

Quiz 3.2: Sample Answers

1. Find the derivative of

$$f(x) = (3x^3 - x - 1) \left(\frac{1}{x^3} + \frac{1}{x^6} \right)$$

We first re-write the function as $f(x) = (3x^3 - x - 1)(x^{-3} + x^{-6})$, then use the product rule:

$$f'(x) = (3x^3 - x - 1)(-3x^{-4} - 6x^{-7}) + (9x^2 - 1)(x^{-3} + x^{-6})$$

Then we expand out the brackets:

$$\begin{aligned} &= -9x^{-1} - 18x^{-4} + 3x^{-3} + 6x^{-6} + 3x^{-4} + 6x^{-7} + 9x^{-1} - x^{-3} + 9x^{-4} - x^{-6} \\ &= 2x^{-3} - 6x^{-4} + 5x^{-6} + 6x^{-7} \end{aligned}$$

2. Find the derivative of

$$f(t) = \frac{-t - 1}{t - 1}$$

We use the quotient rule, then simplify:

$$\begin{aligned} f'(t) &= \frac{(t - 1)(-1) - (1)(-t - 1)}{(t - 1)^2} \\ &= \frac{-t + 1 + t + 1}{(t - 1)^2} \\ &= \frac{2}{(t - 1)^2} \end{aligned}$$

3. Find the derivative of:

$$f(x) = \frac{2x^2 - \frac{x}{e^x}}{x^2 + 2}$$

First, let's find the derivative of $\frac{x}{e^x} = xe^{-x}$, as we will need it when we do quotient rule. Using product rule, its derivative is:

$$x(e^{-x})(-1) + (1)e^{-x} = -xe^{-x} + e^{-x}$$

We then use quotient rule to find the derivative of $f(x)$:

$$f'(x) = \frac{(x^2 + 2)(4x - (-xe^{-x} + e^{-x})) - (2x)(2x^2 - xe^{-x})}{(x^2 + 2)^2}$$

$$f'(x) = \frac{(4x^3 + x^3e^{-x} - x^2e^{-x} + 8x + 2xe^{-x} - 2e^{-x}) - (4x^3 - 2x^2e^{-x})}{(x^2 + 2)^2}$$

$$f'(x) = \frac{8x + x^2e^{-x} + x^3e^{-x} + 2xe^{-x} - 2e^{-x}}{(x^2 + 2)^2}$$